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OPTIMIZATION WITH STOCHASTIC PARAMETERS IN DESIGN OF INDUSTRIAL AIR COOLERS: THE CONCEPT OF SHORTAGE COSTS

Fr. de Pauw

Chemetals Corp., Baltimore, MD, U.S.A., and Catholic University of Leuven, Leuven (Belgium)

C. Lebeer

and

K. Patch

Department of Chemical Engineering, MIT, Boston, MA (U.S.A.)

ABSTRACT

Industrial air coolers are usually designed by assuming a given ambient air temperature, whereas they generally operate under constantly fluctuating outside air conditions. Under these circumstances, minimizing only investment and operating costs on a fixed air temperature basis is not sufficiently accurate to optimize operation. The concept of shortage costs, the opportunity loss of production, can be incorporated in the optimization procedure, in order to take account of the parameters that vary with time. Optimum design specifications are determined accordingly. The air temperature distribution has to be simulated. From the procedure, optimal transfer areas may be calculated which are substantially larger than those obtained with the fixed temperature method.

SCOPE

The specific purpose of design of an industrial cooler consists of determining the optimum equipment characteristics so that a heat load Q can be taken away between two temperatures T_1 and T_2. Until now, it has been usual to minimize the sum of investment and operating costs, on a basis of given or assumed fixed ambient air conditions (temperature, humidity, pressure, solids in suspension, etc.). While every design engineer understands that equipment has to be able to operate under conditions more severe than those used for calculation, the impact of these "abnormal" operating conditions on the optimization of the operation as a whole is usually neglected, and the assumption of the design conditions is left to the judgment and experience of the designer.

This paper examines the impact of daily and seasonal fluctuations of the operating conditions on the optimization of the entire operation, including essentially investment, operation and maintenance costs. In order to account for these operating conditions which are less favourable than the design data, experimental and statistical distributions must

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be available for the optimization procedure. Optimum design specifications are calculated and related to the marginal profit of the manufactured product.

A case study is presented in which shortage costs are introduced whenever the actual air temperature exceeds the design temperature. A Monte Carlo technique simulates the air temperature distribution; a multidimensional secant method solves the design equations of the air cooler and the complex method is used to minimize total costs.

INTRODUCTION

Design of industrial equipment and specially of systems in general is supposed to be an exact science. This implies that calculation is made on the basis of well defined parameters, determined by the environment, be it social, geographical, economic or simply physical. Since every piece of equipment in a system is designed as a function of that system, any change in the operating condition of the system will affect the individual components in their efficiency. Furthermore, since every system is subject to change, on short or long cycle basis, it is necessary to assess the impact of this change on the operation of the system and as a result on its long-range operation.

Changes in economic parameters are already generally introduced into the calculus by admitting a useful lifespan of the equipment, whether this is defined by wear, or by obsolescence, or by abrupt termination of operation, be it by accident, political action, or other. This is then expressed in a depreciation schedule which in turn defines the profitability of the project.

System components are usually supposed to be engineered on a firm set of data which are defined as the "basis of design". It is usually admitted that this basis is not invariant, but that it generally represents, by convention, a reasonable approach of the operating conditions which will be limiting the performance of the equipment. Obviously, this does not imply that these conditions are generally prevalent, nor that they can never be exceeded either. It is the purpose of this paper to present an approach whereby actual operating conditions are simulated over the whole life of the equipment and optimization of the design, as far as sizing, materials selection, maintenance policy, etc., is effected accordingly. In order to do so, traditional optimization methods have to be integrated with an appropriate stochastic input and a cost function that takes into account extreme operating conditions and their probability density.

To obtain stochastic models necessary for design, different approaches have been presented. Berryman and Himmelblau [1] used a Monte Carlo simulation technique to calculate the optimum value of overdesign factors taking into account that a predetermined set of performance criteria must be satisfied, meeting a preselected degree of significance level. Freeman and Gaddy [2] still specified a fixed set of performance criteria but, in addition, optimized the significance level by minimizing the sum of investment costs, operating costs and lost product value.

This paper presents a method to calculate optimum design specifications in the case some of them are themselves stochastic in nature and cannot be predetermined by other than arbitrary selection. Cost items increasing with higher significance levels (e.g. investment costs, energy costs, ...) are balanced against costs reflecting the opportunity loss of production whenever some design specifications cannot be met due to the extreme values which result from their stochastic distribution. These costs are called *shortage costs*: they depend on the actual values of all design variables and are related to the technology and economics of the process. The selection of excessive overdesign factors is thus prevented by using this concept in design, herewith eliminating the drawback of previous methods.

The purpose of the approach is to determine amongst given distributions of design parameters those values that on the long run satisfy the equation:

$$|\Delta \text{ investment costs} + \Delta \text{ energy costs} = \Delta \text{ shortage costs}|$$

which states that the optimal values of the parameter set are obtained when the sum of the marginal investment and energy costs equals the marginal opportunity loss of production. Technical and economic constraints can also be added [3].

PROBLEM DEFINITION

The general problem exposed above will be applied to the design methodology of industrial air coolers.

A fluid must be cooled between the temperatures T_1 and T_2, and a variable heat load Q must be exchanged with the cooling air. The air cooler is assumed to be part of an industrial system, characterized by a production capacity C and a product having a marginal profit *mp* per unit. The latter is equal to the loss of profit when one unit less is produced than target capacity. This represents an "opportunity cost" for the associated process. The value of *mp* can be deducted from the contribution margin of the process, calculated in the cost accounting system of the process.

The characteristic properties of an air cooler are its required area A_0 and the required power for the fan, providing an air flow V. The pressure drop across the pipe bundle Δp is a function of the air flow V.

Air as cooling medium has several desirable features: its overall availability, low fouling tendency and limited corrosion problems. However, a drawback is the fact that the air temperature fluctuates during the year, which must be taken into account during the design stage.

Whereas design in itself on stochastic parameter input is already a subject of considerable attention in literature [4], the optimization procedure has only been treated in very general terms [1,2]. This paper proposes a method to determine optimum significance levels for the random design *variable: the inlet design temperature*. The problem structure is presented in Fig. 1.

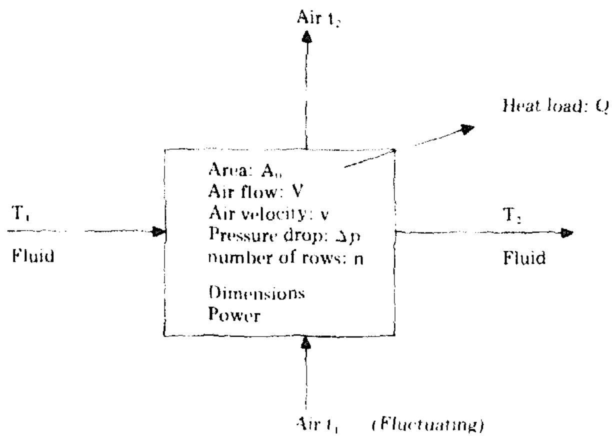

Fig. 1. Problem structure of air cooler design.

The traditional approach is to minimize the sum of the investment costs (*Ic*) and energy costs (*Ec*). The design inlet air temperature t_1 is arbitrarily chosen by selecting a specific degree of uncertainty on the distribution curve of all possible air temperatures for that specific site. A frequently used value is the 5% value, corresponding to an air temperature that was not exceeded more than 5% of the year.

The procedure suggested here provides a methodology relating the location of the optimum design temperature to the marginal profit of the product. It is clear that a very valuable product needs a high design temperature so that no production loss will occur due to undersizing of the equipment. But higher design temperatures will require larger transfer areas and more power, and thus more capital and higher operating costs.

The cost concept associated with a possible reduced production rate is defined as the *shortage cost* [5]. A shortage cost arises whenever actual variables (temperature, pressure, concentration, ...) are such that the variable heat load Q or exit temperature T_2 (or other process parameters) cannot be met.

SHORTAGE COSTS

The calculation of the shortage costs is as follows. For any given heat load Q and any design temperature t_1, a corresponding bare surface A_0 and air flow V are obtained. Then, usual optimization technique establishes:

$$\Sigma\,(Ic + Ec) = \text{minimum}$$

With any set of these values as input, the actual exchanged heat flow Q' can be calculated for every actual air inlet temperature $t_{1\,\text{act}}$ greater than t_1, using the design equations (1)–(9) (see Appendix 1). The shortage costs Sc are then expressed as:

$$Sc = \Sigma p_i\,[mp(1-Q'/Q)C]$$
$$t_{1\,\text{act}_i} \geq t_1$$

In this equation, p_i is the simulated probability of the actual air temperature $t_{1\,\text{act}_i}$ belonging to the air temperature distribution. This distribution is obtained by using daily average temperature interval data of 3°C for the location of Brussels. The data were provided by the Belgian Royal Meteorological Institute.

In order to obtain a distribution curve that would lend itself easily to numerical treatment by a minicomputer, a Monte Carlo simulation has been applied to reclassify the experimental data, with the assumption that all temperatures within an interval of 3°C have the same probability (Fig. 2). Temperatures below −15°C and above 31°C are not considered relevant because of their negligible frequency.

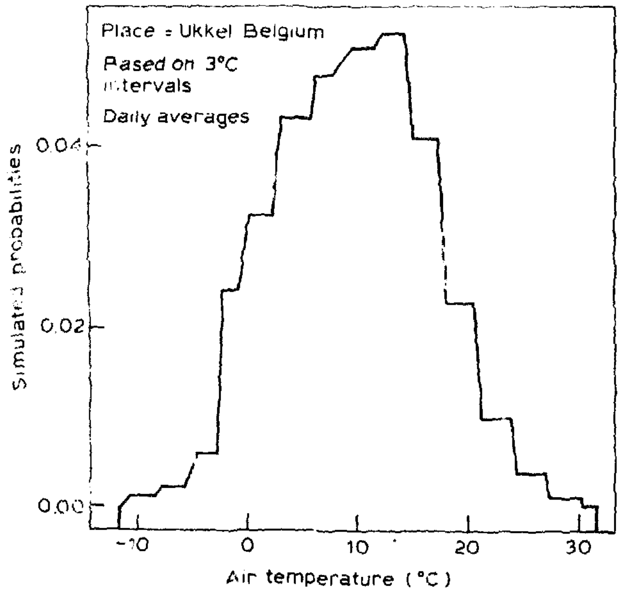

Fig. 2. Air temperature distribution curve.

OPTIMIZATION PROGRAMME

A set of design and cost equations (see Appendices) determines the design and economics of an air cooler. The classification of variables and parameters is shown in Table 1.

The optimization problem consists of determining the optimal set (t_1, Δp) such that the sum of investment, energy and shortage costs is minimized, the design equations (1)–(9) are satisfied as well as the thermodynamic boundary value $t_2 \leq T_2'$. The solution is presented in Fig. 3.

The optimization procedure adjusts during

TABLE 1

Classification of variables and parameters for optimization program

Input data	$h_i, h_f, s, \lambda, y, S, b, l, D_0, L$
air temperature distribution	t_{1i}
Problem parameters	Q, T_1, T_2, mp, C, n
Independent variables to be determined by the optimization	$t_{1od}, \Delta p$
Dependent variables	$t_2, A_0, V, v, Ic, Ec, Sc$

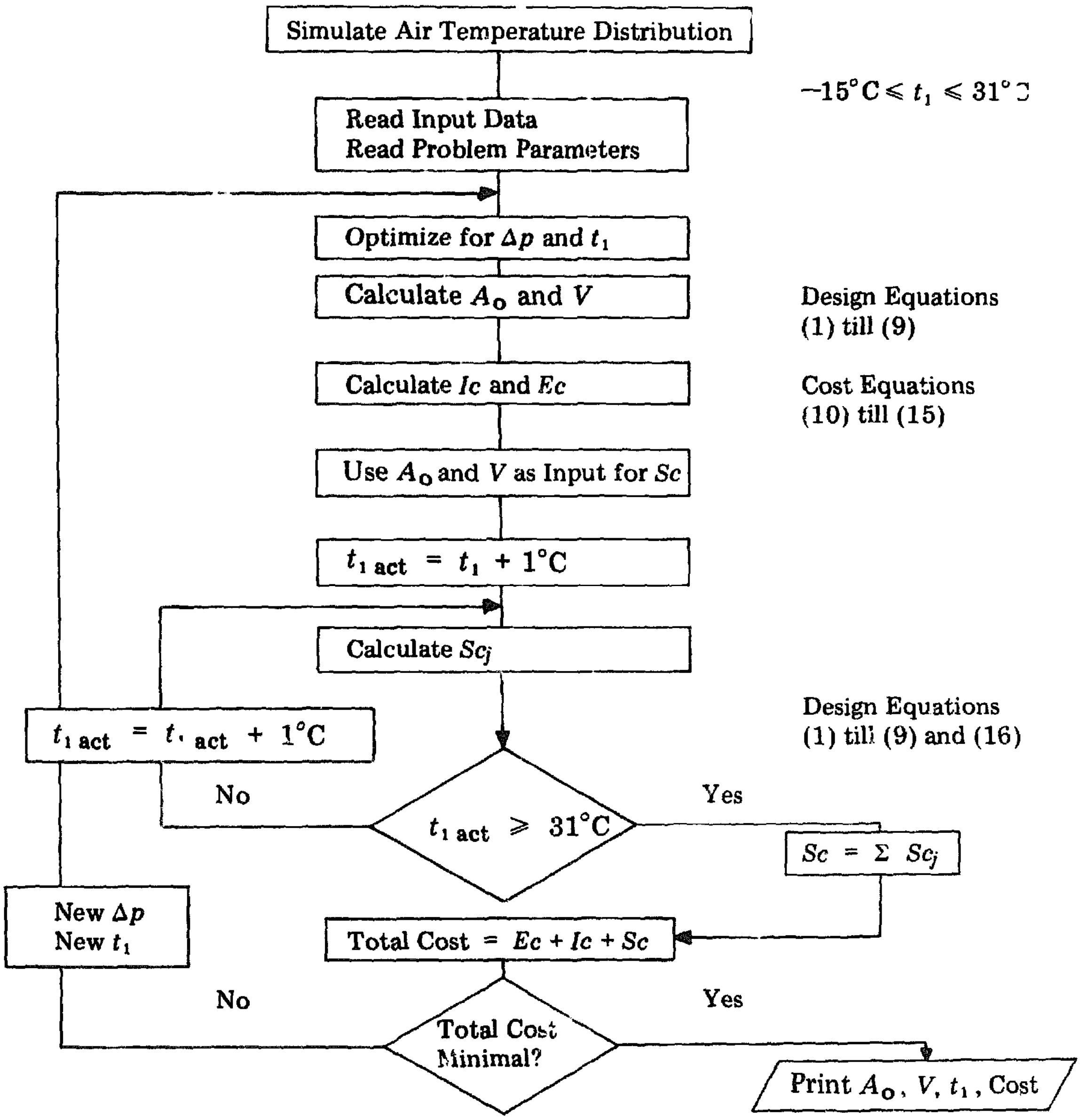

Fig. 3. Program structure.

each iteration step the values of the pressure drop and the inlet temperature to minimize the total costs. The design equations are used to determine the area and the required air flow of the heat exchanger. Once these equipment parameters are calculated, the actual heat flow is obtained by solving the design equations again. The latter is repeated for every actual integer temperature greater than the originally assumed design temperature.

The complex method of Box [6] is selected as optimization technique. A user-supplied initial feasible point which satisfies all explicit and implicit constraints is used to randomly generate an initial simplex of points. The algorithm then proceeds to move the "swarm" of points towards the minimum of the objective function. Convergence criteria include a combination of absolute and relative error terms of the objective function. The multivariable secant method is used to solve the set of nonlinear design equations.

RESULTS AND DISCUSSION

The usefulness of this optimization strategy was tested in the case of air coolers in a chemical plant. Specific data were provided by Carbochimique [7].

A heat load Q of 2×10^6 J/s has to be taken away between the temperatures 42°C and 32°C. The process capacity is 300 000 tons/year and the contribution margin is \$100/ton. Assuming the fluid to be water at 1 m/s, the Sieder–Tate equation gives a value of 4700 W/m² °C for the inner heat transfer coefficient.

The main program output is the location of the optimal inlet air temperature to take for design. The program can also be executed for numerous sets of parameter values. The most important sensitivity studies performed are:
(1) sensitivity of the contribution margin *mp*,
(2) sensitivity of the number of rows *n*,
(3) sensitivity of the fluid exit temperature T_2.

Curves relating total costs to the design inlet temperature are plotted in Figs. 4 and 5. It is clear that an optimal design temperature exists, although in a restricted temperature zone the total profit is rather insensitive to variations in the values of the inlet temperature chosen for optimal design. Hence, there is no need any more to select a design temperature arbitrarily. It has to be remembered that the points on these figures are not generated in the order of increasing inlet temperatures t_1 during the execution of the optimization programme and that the pressure drop varies from point to point.

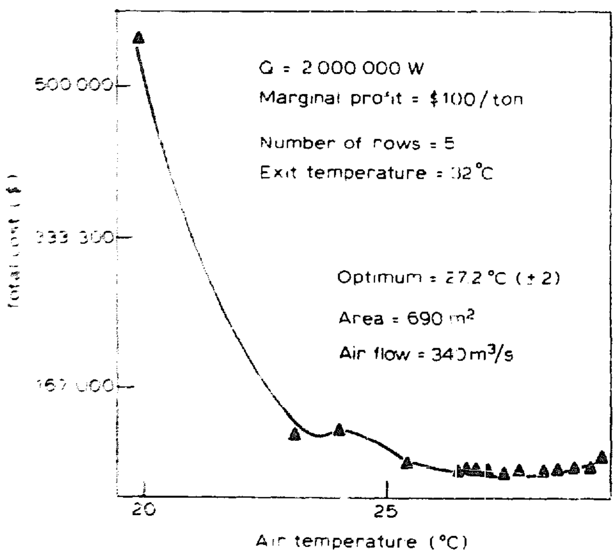

Fig. 4. Total costs versus inlet air temperature selected for optimal design t_{iod}.

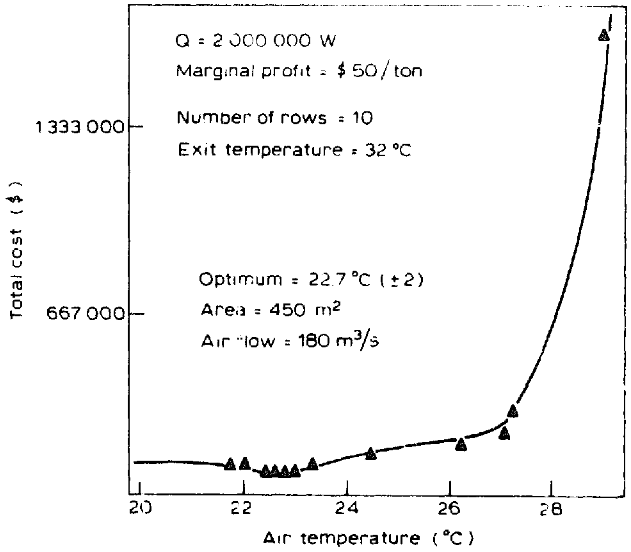

Fig. 5. Total costs versus inlet air temperature taken for optimum design t_{iod}.

Sensitivity analysis of the marginal profit, *mp*

The curve relating inlet temperature air for design to marginal profit is shown in Fig. 6. It should be noticed that the design temperature t_1 increases with increasing product value: production losses then become more substantial. In Table 2, indications seem to point to the fact that a higher design temperature does not necessarily correspond to a larger heat transfer surface. An explanation for this apparent abnormality is that variations in capital and energy costs compensate each other in the numerical calculation of the optimum.

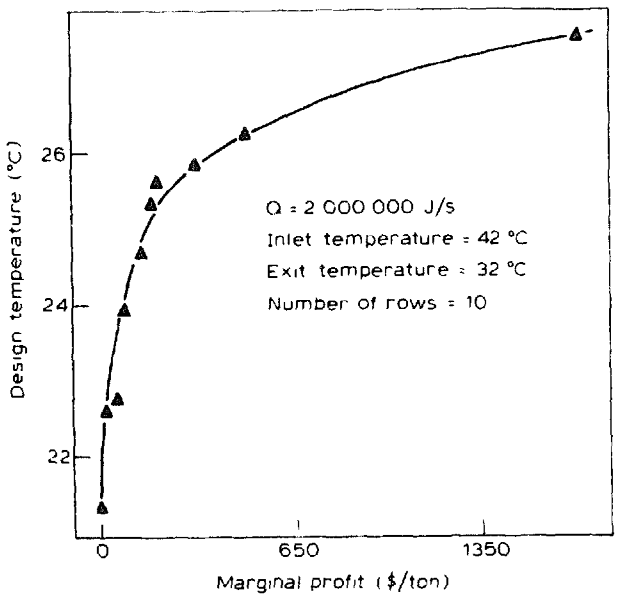

Fig. 6. Impact of marginal profit on design temperature.

When shortage costs are not taken into account ($mp = 0$), the required heat transfer area is 16% underestimated and the air flow more than 25%. The fact that the curve flattens after a certain marginal profit is important. When the product value is low, the three cost terms are of the same order of magnitude; at higher marginal profits, the shortage costs become completely dominant: total costs increase almost linearly with marginal profit.

Total installed power is also a function of marginal profit. When mp equals $100/ton, already a 140 kW fan is required, resulting in a 260% overdesign factor!

TABLE 2

Sensitivity of the optimum design to the marginal profit mp

mp ($/ton)	Total cost ($/year)	t_{iod} (°C)	Installed power (kW)	A_O (m²)	V (m³/s)
0	–	21.2	–	395	150
50	40 000	22.7	30	450	180
100	120 000	24.1	140	468	211
150	150 000	24.6	120	440	226
200	215 000	25.6	200	447	260
500	275 000	26.2	300	454	292
1.650	530 000	27.5	500	482	372

$Q = 2 \times 10^6$ J/s; $T_1 = 42°C$; $T_2 = 32°C$; $n = 10$.

Sensitivity analysis of the number of vertical rows in pipe bundle, n

The number of rows mainly determines the pressure drop of the air flow across the pipe bundle (eqn. (7)). A smaller number of rows implies a larger heat transfer area to trade-off against a lower energy cost. The results are presented in Table 3.

To minimize total costs, a low number of rows should be favoured as done in traditional air cooler design. However, when n is lowered, the space requirements of the equipment, particularly the width, will increase hyperbolically. This is a very site-specific problem where space considerations must be balanced against total costs.

TABLE 3

Sensitivity of the optimum design to the number of rows

n	t_{iod} (°C)	A_O (m²)	Δp (Pa)	V (m³/s)	Total cost ($/year)
3	28.3	965	26	509	66 000
4	27.7	833	46	389	64 000
5	27.2	690	88	342	69 000
6	26.4	614	152	300	82 000
10	24.1	468	545	211	130 000

$Q = 2 \times 10^6$ W; $mp = 100$ $/ton; $T_1 = 42°C$; $T_2 = 32°C$.

The standard air cooler has four rows, a width of 5 m, and a heat transfer area of 122 m² (Appendix 2). When shortage costs are introduced, a heat transfer area of 468 m² is required (Table 3). Specifically, however, the air cooler can either have four rows but a width of approximately 18 m, or be characterized by ten rows and a width of 5 m. Intermediate solutions are also possible.

Brown [8] presents a table which contains optimum heat transfer areas of air coolers as a function of a set of geometric parameters.

The results were obtained by minimizing only investment and operating costs. An air cooler having a width of 5 m, a tube length of 9.14 m and four rows would have, according to Brown, an area of approximately 240 m^2. This value is twice the area of the standard cooler. It is now clear that deterministic optimization procedures could result in seriously underestimating the required area. When the pressure drop and the air flow are multiplied in Table 3, and assuming a fan efficiency of 75%, a four-row heat exchanger requires a horsepower of 40 kW and a ten-row heat exchanger even 140 kW. Compare this result with the 52 kW horsepower of the standard cooler. When a low number of rows can be selected, substantial energy savings result.

Sensitivity of fluid exit temperature, T_2

The fluid exit temperature T_2 is largely specified by process requirements, antipollution regulations, materials handling considerations or other restrictions. Total cost obviously increases when T_2 is lowered. The interesting feature of this is that if a similar analysis is made for other cooling modes (e.g. water cooling, ...), the breakeven temperature can be located where one cooling mode becomes more favourable than the other. This procedure allows management to select the most appropriate cooling system for a particular process.

CONCLUSIONS AND SIGNIFICANCE

The concept of shortage costs can be used to determine optimum design specifications for equipment subject to stochastically varying conditions.

The location of the optimum inlet air temperature on the distribution curve to be used for the design is essentially dependent on the contribution margin of the product. In most cases, it is very different from the arbitrarily chosen design air inlet temperature. Substantially larger transfer areas may be required than those obtained with traditional design methods, where air inlet design temperatures are often taken as a fixed rather than as a varying parameter and consequently often underestimated.

To lower energy costs, the number of rows should be kept to a minimum, as long as space availability permits. Plotting total costs for different cooling modes as a function of fluid cooling temperature permits selection of the most adequate system.

APPENDIX 1

Design equations

The heat transfer between cooling air and the fluid is quantified with the heat flow equation for single pass cross-flow:

$$Q = EUA_0 \Delta T_{\ln} \tag{1}$$

The cross-flow efficiency E is calculated with two-dimensional interpolation, using tables presented in Jacob [9]. The overall heat transfer resistance is the sum of thermal resistances at the inner and the outer wall, the pipe wall resistance itself and thermal losses due to fouling and scaling of the pipe wall. The overall heat transfer coefficient U is obtained with:

$$\frac{1}{U} = \frac{1}{h_i}\frac{A_0}{A_i} + \frac{S}{\lambda}\frac{A_0}{A_m} + \frac{1}{h_f}\frac{A_0}{A_w} + \frac{1}{f}\frac{A_0}{A_i} \tag{2}$$

The heat is actually transferred through the fins and the external surface of the pipes. The total tube area available for heat transfer A_w is:

$$A_w = eA_1 + A_2 \tag{3}$$

A_1 is the fin surface per unit pipe length and A_2 is the external pipe surface without fins, both dependent on the number of fins per unit pipe length. They can be expressed as:

$$A_1 = \frac{1}{4}\pi(D_f^2 - D_0^2)2y \tag{4}$$

and

$$A_2 = \pi D_0(1 - yb) \tag{5}$$

The fin efficiency ϵ is primarily determined by the fin geometry, the thermal conductivity of the material and the thermal gradient in the fin. A calculation procedure for flat circular fins is presented in ref. 10. The approach of Ganapathy [11] is followed to relate the heat transfer coefficient h_f at the fin side to the Reynolds number, the Prandtl number and the dimensional characteristics of the fin. He proposes the following equation:

$$Nu = \frac{h_f D_0}{k} = 0.134 Re^{0.681} Pr^{1/3} (S/h)^{0.2} (S/b)^{0.113} \tag{6}$$

The fin dimensions, S, h and b are respectively the distance between the fins, the height and the width.

Several equations modelling the pressure drop Δp across a bundle of finned tubes have been reported. To simplify calculations, parameters describing specific tube bundle geometry have been omitted. A rather general equation has been suggested by Lohrish [12]:

$$\Delta p = 12.54141 \frac{n(\rho v)^{1.75} \mu^{0.25}}{\rho} \tag{7}$$

In this equation, v represents the air velocity through the smallest cross-sectional area of the tube bundle. The free cross-sectional area A_F is the total area available for air throughput. The latter is expressed as:

$$A_F = \frac{A_0}{n\pi D_0} \tag{8}$$

We can now relate the exchanged heat Q to the temperature increase of the air. Writing a heat balance for the air and assuming isobaric flow gives:

$$Q = \rho c_p v A_F (t_2 - t_1) \tag{9}$$

To simplify the design example, a standard air cooler is assumed, having typical equipment and fin dimensions. Data of several manufacturers were investigated, as well as Kern's handbook [13]. The dimensions used in this study are presented in Appendix 2.

The investment and operating costs

The initial capital outlay of an air cooler is largely determined by the total required area. Jelen [14] uses cost capacity factors to relate the *FOB* costs of industrial equipment to its most important characteristics. For air coolers it is the so-called bare surface A_0 that is the main determinant of capital investment. The initial equipment purchase cost Pc is given by:

$$Pc = Pc_{ref}(A_0/A_{ref})^X \tag{10}$$

The erection of the air cooler has still to be taken into account. During installation direct material costs (fitting, foundations, instrumentation, etc.), direct labor costs and indirect overheads must be added. Guthrie [15] introduces the concept of equipment installation cost ratio. The initial investment is multiplied with a factor to obtain the total installed cost I_0:

$$I_0 = Ir\ Pc \tag{11}$$

The total investment must be converted to an equivalent yearly investment. The present value PV of the investment is calculated, taking into account the annual tax credits resulting from depreciation. The transformation into a constant annualized cost Ic can be done using an annuity factor AF:

$$PV = I_0 - I_0 t\left(\frac{D_1}{1+i} + \frac{D_2}{(1+i)^2} + \frac{D_3}{(1+i)^3} + \ldots + \frac{D_{10}}{(1+i)^{10}}\right) \tag{12}$$

$$Ic = AF\ PV \tag{13}$$

Assuming a ten-year straight-line depreciation, a tax rate t of 48% and an after-tax capital cost of 10%, this becomes:

$$Ic = 0.115 I_0 \tag{14}$$

The cost of capital can be calculated with the *CAPM* model or with formulas based on the Miller–Modigliani theorems.

The energy costs Ec arise from the electric motor drive for the fan. They can be derived from the Bernouilli equation for the flow of incompressible gases.

$$Ec = \frac{V(\Delta p + \rho v^2) H P_E}{1000\eta} \tag{15}$$

A good average value for the efficiency of a representative air cooler fan under normal operating conditions is needed. Glass [16] suggests a value of 75%. The load factor H is rather arbitrarily taken as 8000 h/year, representing a 30 days plant shutdown mainly for maintenance of the process. The unit power cost was taken at 7 ¢/kWh.

APPENDIX 2

Determination of standard air cooler characteristics

Data of several heat exchanger manufacturers were investigated, as well as Kern's handbook [13]. All dimensions and characteristics were checked with Carbochimique [7].

Heat exchanger

Base area = 10.5 × 2.5 m
Two elements
42 pipes
4 rows
2 motors of 26.1 kW each

Fin characteristics

Flat circular fins; L feet
Height = 0.0127 m
Thickness = 4.2×10^{-4} m (on 1/2 height)
Distance between fins = 2.3×10^{-3} m
Material: aluminum
Total transfer area/bare surface: 15.5

Pipes

Triangular pitch; transverse pitch equals longitudinal pitch
Outside diameter = 0.0254 m
BWG = 1.66×10^{-3} m
Length = 9.14 m
Material: carbon steel

APPENDIX 3

Calculation of equipment purchase cost correlations

The purchase cost *FOB* of an air cooler is related to the bare surface A_0, according to Woods [17]:

$$Pc = 12\,000(10.76 A_0/1000)^{0.8}$$

The conditions for this equation to be valid are:

- finned pipes; pressure = 1 atm; carbon steel; length = 4.8 m;
- total transfer surface/bare surface = 16 m^2/m;
- included: pipe bundle, heads, fans, explosion-proof motor;
- not included: foundations, direct installation cost, testing, transportation, general overheads.

The equipment cost consists mainly of labor cost and direct material cost. The following assumptions are made to obtain the capital cost correlation in Belgium:

- equipment index 1976–1979: 30%;
- labor cost/total cost: 50%;
- labor cost in Belgium is 60% higher than in U.S.A. (1980 currency rates);
- materials cost in Belgium is the same as in U.S.A. (1980 currency rates);
- 15% cost increase to increased pipe length.

The correlation (10) becomes:

$$Pc = 20\,280(10.76 A_0/1000)^{0.8} \tag{10'}$$

The equipment installation cost ratio is derived from Guthrie [15]:

$$I_0 = 2.3 Pc \tag{11'}$$

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NOMENCLATURE

A_o	bare surface (m^2)
A_i	inner pipe surface (m^2)
A_F	free cross-sectional area (m^2)
AF	annuity factor
A_1	fin surface/pipe (m^2/pipe)
A_2	external surface of pipe without fins (m^2/pipe)
C	capacity of process (ton/year)
$D_1, \ldots, D_i$	amortization factors in years $1, \ldots, i$
D_f	pipe diameter (fins included) (m)
D_o	outside pipe diameter (m)
D_i	inner pipe diameter (m)
E	cross-flow efficiency
Ec	energy costs ($)
H	number of operating hours per year
I_o	total investment ($)
Ir	equipment installation ratio
Ic	yearly capital investment ($/year)
L	pipe length (m)
P_E	price per unit power ($/kWh)
Pc	equipment purchase cost ($)
Q, Q'	heat flow (W)
S	distance between fins (m)
Sc	shortage costs ($/year)
T_1, T_2	inlet and outlet temperature of fluid (°C)
U	overall heat transfer coefficient (W/m² °C)
V	air flow (m^3/s)
b	fin thickness (m)
f	fouling coefficient (W/m² °C)
h	fin height (m)
h_f	heat transfer coefficient at the fin side (W/m² °C)
h_i	heat transfer coefficient at the inner side (W/m² °C)
k	air thermal conductivity (W/m °C)
l	distance between pipes (m)
mp	marginal profit ($/ton of end product)
n	number of rows
p_i	probability density of temperature t_i
Δp	pressure drop across pipe bundle (Pa)
s	wall thickness (m)
t_1, t_2	inlet and outlet air temperature (°C)
t_{iod}	inlet air temperature selected for optimum design
t	tax rate
x	cost capacity factor
y	number of fins per unit length (m^{-1})
v	air velocity (m/s)
ϵ	fin efficiency
λ	thermal conductivity of pipe material (W/m °C)
μ	viscosity of air (kg/m s)
η	motor efficiency
ρ	air density (kg/m^3)

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